

# Spherical image analysis for folding templates

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## Abstract

Origami structures are replete with vertices where fold lines meet and intersect. Assuming inextensible folding, we can solve for vertex kinematics graphically using Gauss' construction of a unit sphere. Each vertex is made up of several facets whose relative folding is captured by the outline shape of a spherical image on the unit sphere: its side lengths are equal to fold line rotations, and the net area enclosed is zero because of inextensibility (following Gauss' famous theorem).

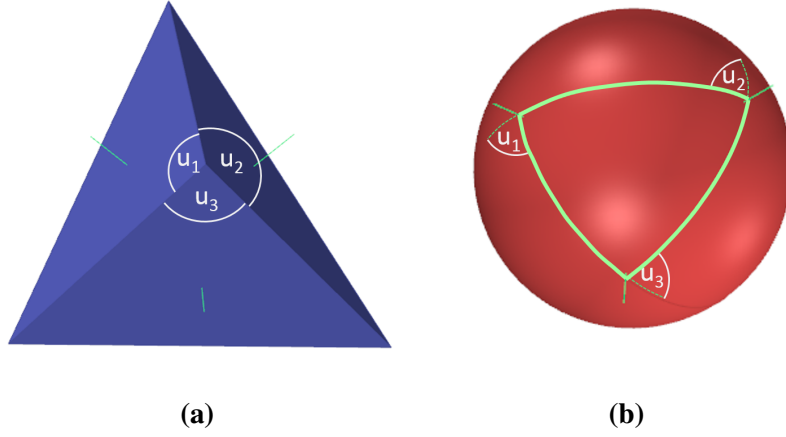
Small fold line rotations result in spherical images that are approximately flat with straightforward geometries. Larger rotations demand more accurate spherical trigonometry, Fig. 1, which is more complex, and daunting, and not often amenable to final, compact expressions in closed form. Consequently, spherical image solutions of large vertex folding are typically numerical and bespoke, where each case is treated on its own merits.

In this work, we provide *equivalent* spherical images for a range of vertex layouts. We begin by classifying vertex types topologically by analogy to bar frameworks. For each type, we can furnish a kinematical description based on its unique spherical image. An arbitrary vertex can be thus systematically parameterised.

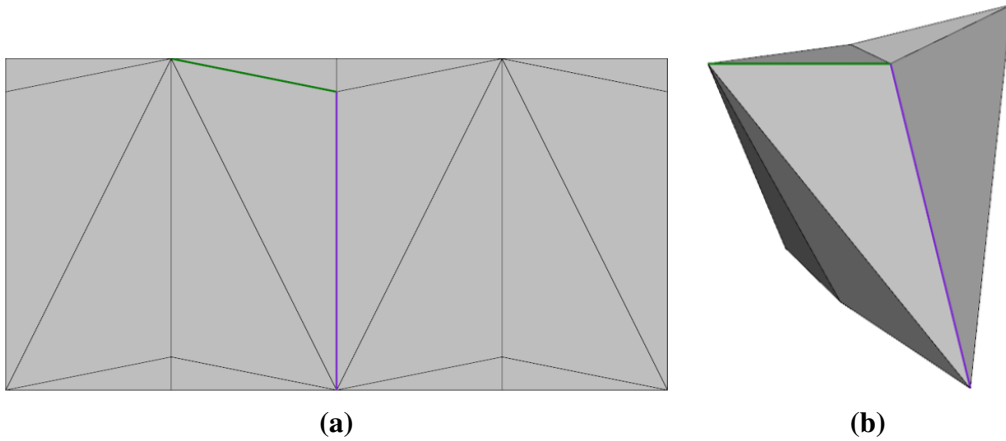
A major application and demonstration of the method concerns the folding of flat sheets into fully enclosed polyhedral volumes: although not allowed under Origami rules, we would stitch the enclosing edges together, in order to rigidify the final shape.

Each flat sheet begins as a parameterised template of fold lines and reconnecting edge lines, Fig. 2; we can adjust the final shape by altering their distribution. Nodal intersections correspond to prospective vertices, where fold line rotations not only satisfy inextensibility but also a condition of complete enclosure, which is easily addressed by our method.

In particular, we are interested in maximising the folded volume attained from an original rectangular sheet. Several classes of shape are considered and we normalise our results with respect to a spherical volume of the same surface area: we achieve comparative volumes of 75.7%. This is a remarkable result given that any spherical enclosure from a flat sheet demands outright, and unfavourable, stretching. We also use symmetry to reduce further the amount of calculation required, neatly demonstrating the elegance of this technique.



**Figure 1:** (a) The intersection of three fold lines, separated by three flat triangular facets to make a roof vertex. The outward facing surface normal are shown in green. (b) Spherical image of the roof vertex, surface normals and the connecting arcs are shown in green.



**Figure 2:** (a) An example of a template which folds up into an enclosed volume. The fold lines about which the template will be folded are shown. Green and purple fold lines are highlighted to aid visualisation. (b) The enclosed volume which the template folds up into. The geometry is non trivial but is entirely determined from the fold line pattern. Calculating the geometry of this shape and can be tackled efficiently using a spherical image based analysis of the vertices.